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Report of the leading organization on the PhD thesis of Erik Khastyan entitled "Supersymmetric mechanics with Kähler phase space"

The studies of Kähler manifolds constitute one of the important traditional subjects of differential geometry and mathematical physics. Concerning mathematical physics, this is due to the pivotal role these manifolds play for the description of moduli spaces in supergravity theories, for models of string compactifications, as well as for the theory of dynamical systems. Kähler manifolds have an extended number of symmetries which in many cases allow for a development of powerful geometric tools to solve dynamical systems defined on these manifolds.

Even though a number of interesting integrable systems come with phase spaces endowed with the Kähler structure, the favorable consequences of this observation do not seem to have been fully explored. The aim of the present thesis is to partially fill this gap and to further contribute to the study of Kähler manifolds as phase spaces of integrable dynamical systems, in major, by constructing the integrable supersymmetric extensions thereof. In this quest, the Kähler structure of the corresponding bosonic models is of great help, as the supersymmetric extensions can be naturally built in the framework of the super Kähler geometry.

The thesis consists of five chapters. The first chapter offers an introduction into Kähler geometry, compact and non-compact versions of complex projective spaces, the hamiltonian dynamics on symplectic manifolds, including the case of integrable and super-integrable systems (i.e. Liouville integrable systems that have additional integrals of motion obeying a non-commutative Poisson algebra), and supersymmetric mechanics. This is a helpful material which substantially simplifies the reading of the rest of the thesis.

Chapter 2 deals with historically one of the first integrable systems – the Euler top. Upon fixing the value of the Casimir function, this is a Liouville integrable system on $\mathbb{CP}^1$. Hereafter, Mr. Khastyan applies a supersymmetrization procedure to obtain $\mathcal{N} = 2k$ supersymmetric extensions of the Euler top. In accordance with the generalization of the Liouville theorem to supermanifolds, these supersymmetric extensions are automatically integrable. For each $\mathcal{N}$ the expressions for the supersymmetry generators in terms of a certain number of undetermined functions are given.

In chapter 3 the superintegrable models related to the non-compact complex projective space are studied. These include the generic superintegrable deformations of oscillator and Coulomb systems, presented in terms of the higher-dimensional Klein model realization of the phase space. Integrals of motion for these systems are expressed via Killing potentials that define the $\mathfrak{su}(N,1)$ isometries of the underlying Kähler structure.

In chapter 4 the $\mathfrak{su}(1,N|M)$ superconformal mechanics is formulated on the $(N|M)_{\mathbb{C}}$-dimensional Kähler phase space. By introducing special coordinates, this model is described as a higher-dimensional superanalogue of the Lobachevsky lower half-plane plane (Klein model). Mr. Khastyan introduces the

canonical coordinates that separate the radial and angular parts of the model. Relating the angular coordinates with the action-angle variables, he is able to construct $\mathfrak{su}(1,N|M)$ superconformal extensions of a wide class of superintegrable systems. These include, in particular, the superintegrable oscillator- and Coulomb-type systems with the $\mathfrak{su}(1,N|M)$ dynamical superalgebra. Further, Mr. Khastyan shows that oscillator systems admit the deformed $\mathcal{N}=2M$ Poincaré supersymmetry, in contrast to the Coulomb ones. Finally, chapter 5 is devoted to the discussion of the results obtained and the future research directions.

The results of the thesis are original and made publicly available in four papers published in the international refereed journals Physical Review D, Physics Letters A, International Journal of Modern Physics A, and Physics of Particles and Nuclei Letters, as well as, in two conference proceedings PoS-Regio. They were also presented by Mr. Khastyan as a speaker on a few international conferences and workshops, including those in Dubna, Tbilisi and Yerevan (2022) and Dubna (2023). The methods of supersymmetrization of integrable models developed in the thesis open new interesting directions towards encompassing a wider class of integrable models with Kähler phase space, for instance, the Ruijsenaars-Schneider models with a compact $\mathbb{CP}^N$ phase space, with or without spin degrees of freedom.

Concerning questions to the thesis, the following one can be asked. In chapter 4 an attempt to realize the deformed $\mathcal{N}=2M$ Poincaré supersymmetry for the Coulomb-like systems is presented. An attempt is based on setting up an ansatz for the supersymmetry generators followed by a check of closure of the supersymmetry algebra. Two ansätze are proposed which both fail to provide the closure of the algebraic relations, hence a negative result. The question is to which extent these studies are exhaustive, i.e. why there could not exist another hypothetical ansatz that would lead to a desired closure of the supersymmetry algebra? It would be nice to clarify this point.

The above question does not influence the quality of the thesis in any way. The results obtained are significant and of interest to a wide community of theoretical physicists. The thesis fulfills the requirements expected of a PhD thesis in the field of theoretical physics and Erik Khastyan deserves the PhD degree to be granted.

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